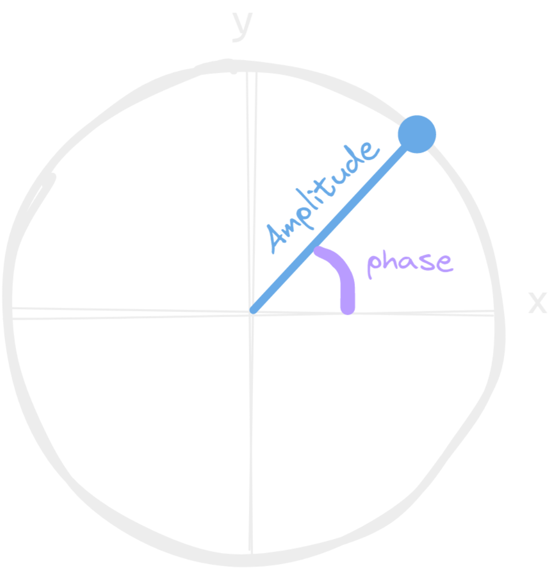
Superposition Pathway

It consists of 5 Topics:

1. Superposition in waves

> Aim is to develop an intuitive understanding of superposition as interference of waves.

- Amplitude and wavelength are independent parameters.
    - For sound, the higher the amplitude, the higher the energy.
    - For light, the higher the wavelength, the lower the energy.
- When waves meet, they do not affect one another. However, the displacement of the medium they are travelling in is affected by both simultaneously. This is called interference.
- A quantum computation can be thought of as coordinating interference to achieve some advantage.
- Three different effects can happen:
    - the waves can both be present, but not near each other
    - the waves can displace the medium in the same direction to constructively interfere
    - the waves can displace the medium in the opposite direction to destructively interfere.
- Wave interference can be seen in action in two dimensions as well, e.g. ripple in a pond.
- The waves don't affect one another, only the medium they travel in.
- Two identical waves added together produce the same wave form with double the amplitude.
- Wave motion and circles have a surprisingly tight connection. Uniform circular motion, constantly moving around a circle, produces a perfect wave.
- The size of the circle is the amplitude; the angle from the positive x-axis toward the positive y-axis is the phase; and the speed relates to the wavelength.



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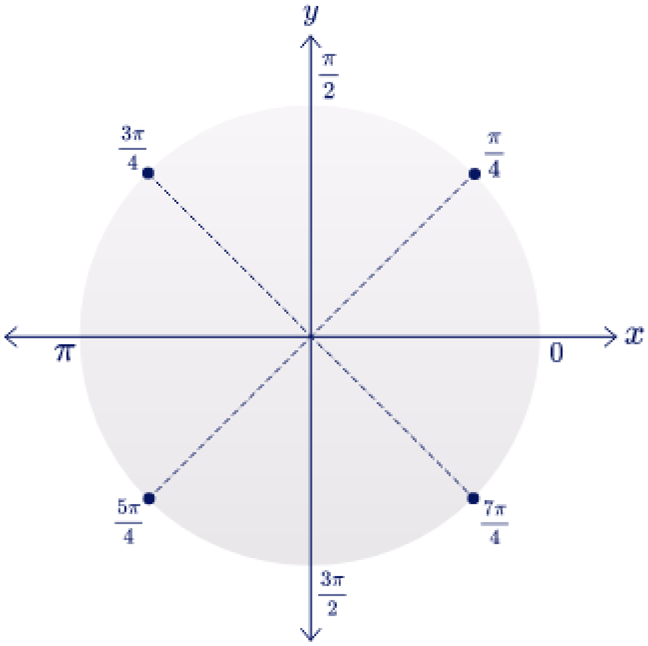
- For most of our journey through quantum computing, the circles will have a radius of 1.
- Some of the handy angles to remember:



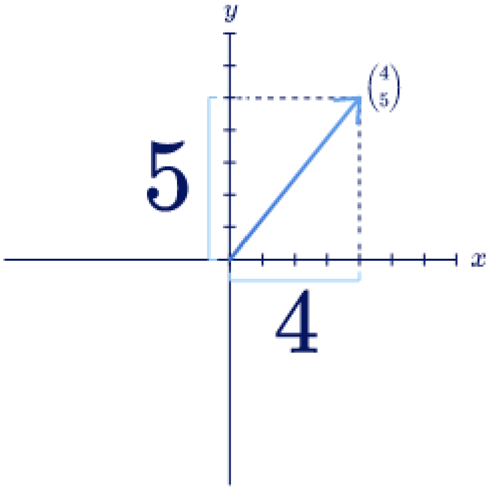
superposition_circle_wave_angles.png

2. *Superposition in a jump rope*

> Move toward a more abstract representation of superposition in the polarization degree of freedom possessed by waves.

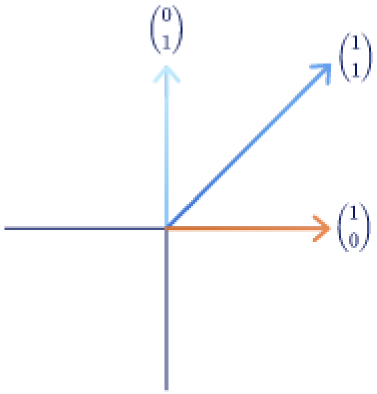
- Any wave that vibrates in one direction only is called a Linear wave. This orientation of its vibration is called Polarization.

- For polarization, a vector measures both the vertical (y) and the horizontal (x) contribution and can be noted concisely as $\begin{pmatrix} x \\ y \end{pmatrix}$.



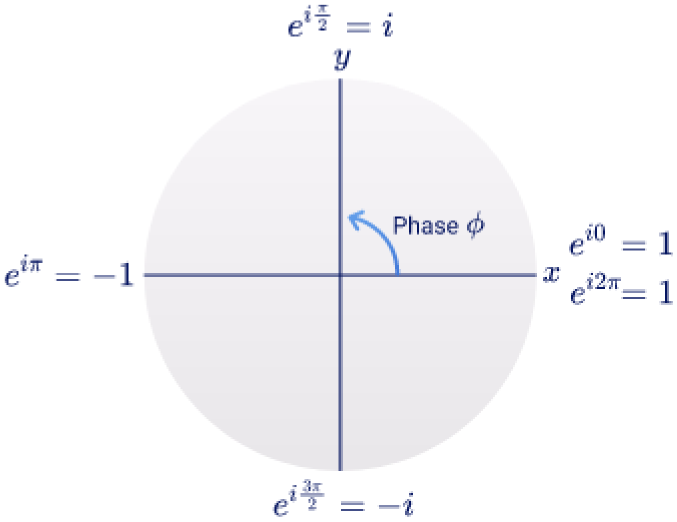
POLARIZATION_DRAWN_AS_VECTOR.PNG

- A vertically polarized wave is represented by a vector like $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
- A horizontally polarized wave is represented by a vector like $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
- Diagonal polarization has a vector like $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$
-



POLARIZATION_VECTOR_REPRESENTATIONS.PNG

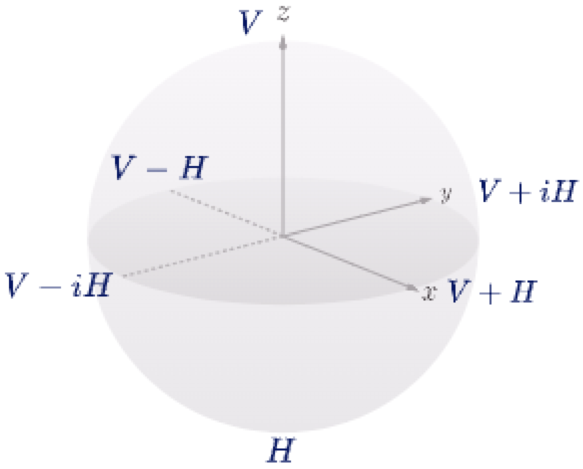
- The six special polarizations are called
  - vertical
  - horizontal
  - diagonal
  - anti-diagonal
  - right-circular
  - left-circular
- Any polarization can be written as $V + e^{i\phi}H$, where ϕ is the phase and i is the imaginary unit that satisfies the equation $i^2 = -1$. Here V and H represent vertical and horizontal polarizations, respectively.



SUPERPOSITION_POLARIZATION_VALUES.PNG

> - Diagonal polarization: $V + H$
> - Anti-Diagonal polarization: $V - H$
> - Right-Circular polarization: $V + iH$
> - Left-Circular polarization: $V - iH$

- Just as diagonal and anti-diagonal superposition waves are combinations of V and H, vertical and horizontally polarized waves can be thought of as superpositions of diagonal and anti-diagonal waves.
- All possible polarization states can be pictured at once with a sphere. Specifically in the context of polarization, this is called the [Poincaré sphere].



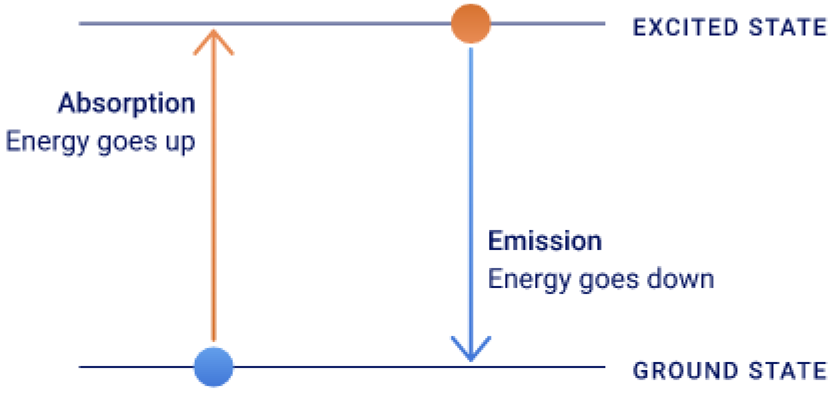
POINCARE_SPHERE_VISUALIZATION.PNG

- In quantum computing, vertical and horizontal polarizations are labelled as 0 and 1, and these form the computational basis of a quantum bit — a Qubit!

3. Superposition in the quantum world

> Superposition in example quantum systems relevant to quantum computing: ions, atoms, superconducting circuits, photons, and more!

- In quantum physics, we discovered that Energy is quantized, meaning it is observed to be discrete. These discrete values of energy are called Energy levels — the steps of energy any thing can be measured to have.
- The state of lowest possible energy a system can have is called the **ground state**. All other states the system can be in are called **excited states**.



ENERGY_LEVEL_STATES.PNG

- Typically in quantum technology, only two of the energy levels are used to store quantum information, though other energy levels can assist in processing it.
- A quantum system using two quantum states, and superpositions of them, is called a Qubit, or quantum bit.
- While most digital technology has converged on one physical realization of the bit (a semiconductor transistor), quantum technology has many qubit realizations which are candidates for a scalable architecture.
  - **Trapped ions**: An ion is an atom with more or fewer electrons than protons, and hence a net electric charge. The net charge of the atom allows it to be trapped using electric forces.
    - Once a single atom is trapped, the valence electron's energy can be manipulated using lasers. Any two conveniently chosen energy levels are distinguishable with lasers and detectors. Whatever they may be, they are labelled with the ket notation $|0\rangle$ and $|1\rangle$ and used to define a qubit.
    - Thus, in an ion trap quantum computer, superpositions are those of linear combinations of energy states of individually trapped ions. An example of a superposition state is $\frac{3}{5}|0\rangle + \frac{4}{5}|1\rangle$.
  - **Atoms**: These are isolated individual Neutral atoms, not susceptible to electromagnetic forces. They require different trapping techniques from Trapped ions.
    - Multiple laser beams intersecting a single point can be used to hold an atom in place. The lasers form a pattern of "wells" for the atoms to sit in.
    - Once the atoms are trapped, their internal energy can be manipulated with lasers, much like trapped ions.

    - To define a qubit, two energy levels that can be easily addressed are chosen to act as the $|0\rangle$ and $|1\rangle$ states.
- **Color centers**: When atoms arrange themselves in regular, repeated patterns they create a crystal. In a crystal, no part is different from the rest, except if there is a defect. A defect may be a missing atom or a different kind of atom that doesn't fit the pattern. The locations containing a defect are more generally called color centers (e.g. defects in a diamond crystal can give it color).
    - In some cases, a defect can be occupied by an electron. The electron is localized if the defect can be found!
    - The energy of the electron can then be changed and observed by focusing lasers at the site of the defect.
    - Two energy states are chosen to define the qubit, and thus linear combinations of those energy states can be created.
- **Electrons in semiconductors**: Electrons in atoms are not localized but exist in more vaguely defined orbitals. Thus, Directly accessing the energy states of electrons is extremely challenging.
    - With advances in semiconductor technology, manufactured devices can trap electrons using a variety of competing techniques usually referred to by the name of the material being used (e.g. silicon or gallium arsenide). These devices are placed as components into microelectronic circuits and electronic pulses are passed through them to access their energy states.
    - A common example is the internal spin of an electron, which is an ideal realization of a qubit since it possesses exactly two states, called spin-up $|\uparrow\rangle$ and spin-down $|\downarrow\rangle$.
    - These spin qubits have superposition states which are linear combinations of spin-up and spin-down.
    - In the context of computation, these are usually relabeled to the more familiar $|0\rangle$ and $|1\rangle$.
- **Superconducting circuits**: Circuits made from Superconductor material can be designed to have unique energy level structures — designer artificial atoms, which define qubits. These go by equally exotic names, such as charge qubits, flux qubits, and phase qubits.
    - There are many differences between these circuits, each with its own advantages and disadvantages. However, the one thing they have in common is that the quantum information is contained in two nearby low-energy states.
- **Individual photons**: Photons can encode qubits in the polarization of the electromagnetic wave that they carry.
    - Two different states that can be distinguished are horizontal and vertical polarization. A superposition of these states is a new polarization.

    - For example, an equal superposition of horizontal and vertical is diagonal polarization.
  - Depending on the convention, or reference frame, either horizontal or vertical polarization is given the label $|0\rangle$. The other would be labeled $|1\rangle$.

4. Superposition in the abstract

> Superposition of abstract objects representing quantum systems.

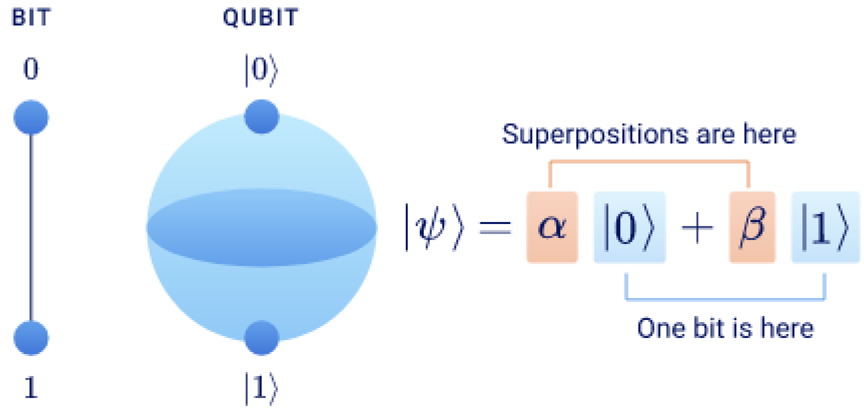
- The focus on energy states is a choice made by us, rather than Nature.
- We can use Arithmetic to represent an abstract view of superposition.
- Ket $|\psi\rangle$ is a vector that occupies an abstract complex vector space.
  - First, the word complex means that superpositions can have complex numbers in them instead of only *real* numbers. (phases)
  - A vector space is a set of objects that can be added together in superposition and satisfy some other simple rules. It's abstract because there are lots of examples that satisfy the rules in the definition including simple sets of objects like numbers.
  - Finally, a vector is one of the objects in the set.
- The state is the way something is — a complete description of all that is known about it.
- Quantum states are states that may represent quantum information.
- With them we can discuss quantum computation at the abstract level without needing the details of the physical device that will do the work.
- The smallest unit of quantum information is called a quantum bit, or Qubit.
  - A qubit describes a two level system.
  - Those two basis states define our vector space: $|0\rangle$ and $|1\rangle$, which can of course be represented as rows of numbers: $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$.
  - These can be added together in any complex superposition - that means we can make a sum with a complex number in front of $|0\rangle$ and another complex number in front of $|1\rangle$.
  - Another constraint says that both complex numbers need to have a total "length" of one.
- This can be visualized using a Bloch sphere.

5. Superposition in quantum computing

> Superposition in a quantum computation and quantum parallelism.

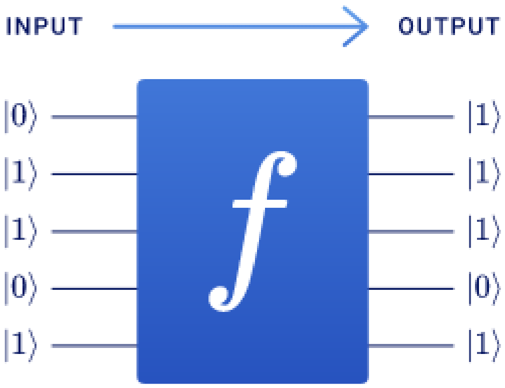
- Any piece of information can be thought of as the answers to a series of yes-or-no questions.

- The smallest unit of quantum information is the Qubit. Multiple qubits encode generic quantum information. A qubit can encode the answer to a single yes-or-no question, but a qubit also contains richer information thanks to superposition!



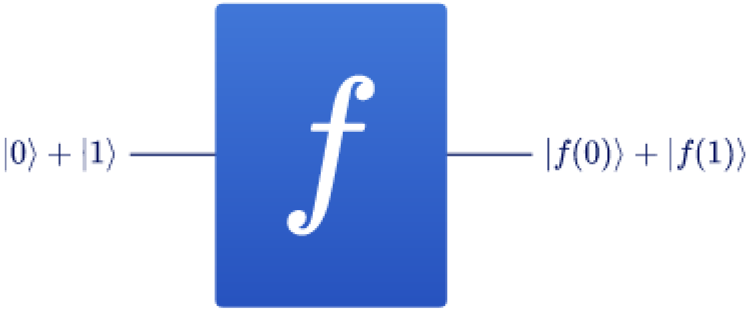
BIT_QUBIT_INFORMATION.PNG

- The power of quantum computation lies not in how much information can be stored in qubits, but how few steps are required to solve problems.
- The NAND function is a universal function - any computation can be done by a sequence of NAND functions - made by combining the NOT function and the AND function.
- A convenient way to look at small quantum computations is with a quantum circuit. The lines on the circuit diagram hold the quantum states and are called wires or registers.



QUANTUM_CIRCUIT_DIAGRAM.PNG

- A quantum circuit acts on each part of a superposition.



QUANTUM_FUNCTION_SUPERPOSITION.PNG

This is known as quantum parallelism or superposition of data.
- Reading quantum information destroys superpositions. The clever way around this problem is to create not simply a superposition state, but an interference pattern.

Qubit Pathway

> Start with some fundamentals in how we represent information and by the end of this skill you'll understand how we can encode and process information using quantum physics!

It also consists of 5 topics:

1. It from bit

> This topic introduces you to the concept of information. You will learn how information is defined and represented, as well as how it is measured and used.

- There are three fundamental types of quantitative data: analog, digital, and quantum. These data types are quantitative in that they use numbers to represent information.
  - Analog data uses real numbers, which change smoothly and continuously.
  - Digital data uses integer numbers, which can be counted and change discretely.
  - Quantum data uses complex numbers, which adds a new dimension with sometimes unintuitive consequences.
- Analog data is prone to errors while digital data entails a loss of information.
- Quantum data is the best of both worlds. While the data itself is prone to error (like analog data), with the right techniques it can also be protected against error (like digital data).
- Quantum data is single use i.e. it can not be copied. Once it is read, it collapses and is gone.
- The process of changing physical things into configurations representing data is called encoding.
- Leading candidate technologies for quantum data encoding include trapped ions or neutral atoms, superconducting circuits, defects in crystal structures, and photonic circuits.

2. Qubit from bit

> This topic introduces quantum information. You will learn how quantum information is written and read and how it is different from digital information.

- Whereas the state of a bit is either 0 or 1, the state of a qubit is a point on the surface of a sphere.
- Reading quantum data — a process called measurement — destroys it! The necessity of measurement to extract information from qubits means they are single-use.
- Measurement isn't just restricted to human. Even the surrounding environment of the qubit constantly tries to alter/read the state of the qubit. This is called noise.
- Noise quickly corrupts the state of the qubit and causes errors in computation. This is the biggest challenge in quantum technology.
- The exact center of the sphere is the state of a fair coin, with an equal probability for 0 or 1 — and no quantum features! The other locations on the sphere are states of a qubit representing genuine quantum information.
- Noise randomizes the qubit's state, and sometimes, even turns it back into classical bit.
- No-gos in quantum computing are those things that we cannot do with quantum information but are possible in classical computing.
- There's no-cloning in QC refers to the fact that qubits can't be copied or cloned. Qubits of data are unique, and if you receive them, you can be assured no copies exist.
- The states on the qubit sphere that are not $|0\rangle$ and $|1\rangle$ are called superposition states. Every superposition state is a combination of $|0\rangle$ and $|1\rangle$, and can be written as $\alpha|0\rangle + \beta|1\rangle$. α and β are the constraints to keep the qubit on the sphere.
- The most common superposition state is called the **plus-state** $|+\rangle$. This is an equally weighted sum of $|0\rangle$ and $|1\rangle$: $||+\rangle$ = $|0\rangle + |1\rangle$.
- The second most famous superposition state is the “minus-state”: $|-\rangle = |0\rangle - |1\rangle$.
- Entanglement is most succinctly defined as superpositions of correlation.
- The entire system of entangled qubits must be considered its own enormously complex state of information.
- Entanglement is necessary for quantum computation to yield an advantage over digital computation.
- In many quantum algorithms, it is assumed the qubit is supplied in the state $|0\rangle$.

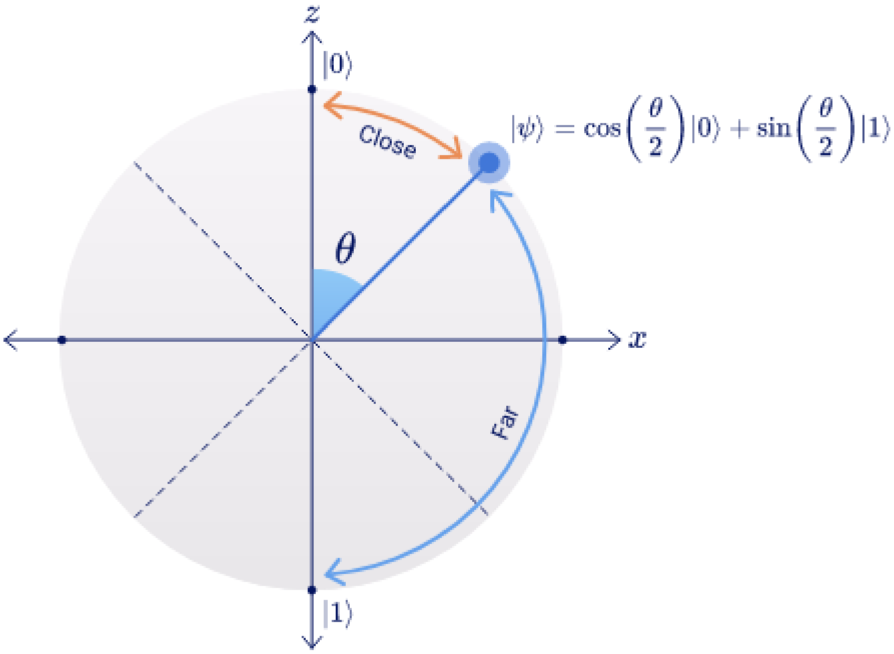
3. The Bloch circle

> You may have heard of the Bloch sphere, and we’ll get there soon. Here you will find a first step towards a visualization of qubits: a circle. You will learn how a qubit is written, read, and processed in a geometric picture.

- We can never observe superpositions as the measurement destroys superposition states and only lets us observe the computational states $|0\rangle$ and

$|1\rangle$.

- This fact does not imply that the outcome of the measurement is arbitrary. The outcome of the measurement is indeed random, but occurs with a probability given by the rules of quantum mechanics.
- A circle is a geometric constraint on length
- The circular constraint, $\alpha^2 + \beta^2 = 1$, is called normalization which is required to conserve the probability. The numbers α^2 and β^2 are the probabilities, which must always add up to 1.
- The numbers α and β are called amplitudes. In classical logic we add probabilities like α^2 and β^2. In quantum logic we add amplitudes.
- We can connect the qubit superposition to the angle in our Bloch circle: $|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + \sin\left(\frac{\theta}{2}\right)|1\rangle$.



AMPLITUDE_ANGLE_TRIGONOMETRY.PNG

- Quantum Gates change the state a qubit from one place to another.
- For a single qubit, there are only two digital logic gates possible: the identity gate and the NOT gate.
- The convention in quantum computing is to call the NOT gate X, which exchanges 0 and 1. This works in superposition as well: $\alpha|0\rangle + \beta|1\rangle \xrightarrow{X} \alpha|1\rangle + \beta|0\rangle$.
- The Hadamard Gate H could be called the superposition gate as well since it acts as $|0\rangle \xrightarrow{H} |+\rangle$, $|1\rangle \xrightarrow{H} |-\rangle$.
- All gates have some special states they have no effect on called eigenstates, e.g. $X|+\rangle = |+\rangle$.
- The Y gate applies a 180° or π rotation about the y axis. Half a Y gate is 90° or $\frac{\pi}{2}$ rotation about the y axis.
- In addition to rotations in the clockwise direction, we can have counterclockwise y axis rotations. This is not “minus Y”, but the inverse of a Y gate, Y^{-1}.

4. Do you $|ket\rangle$ it?

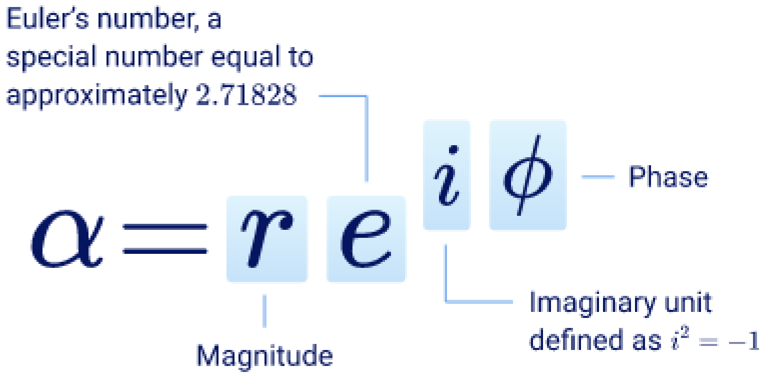
> Kets, like |this⟩, are the abstract representation of qubits. In this brief topic, you will learn what these objects mean and how to use them.

- $|\psi\rangle$ is the abstract symbol for a state of quantum information.
- Kets are symbols used to write down the states of qubits.
- In the digital world, kets are represented as vectors, e.g. $|\psi\rangle = \begin{pmatrix}\alpha \\ \beta\end{pmatrix}$.
- Any state can be written as a set of gates applied to the state $|0\rangle$.
- Gates are *applied* to qubits, i.e. a gate acts on the state written next to it on the right. The order matters.
- The symbol U is used for an arbitrary gate. So, the abstract thing we can write so far is $U|\psi\rangle$.
- Every gate U has an inverse gate, written U^{-1}. Applying U and then its inverse is written $U^{-1}U = I$. The gate I, called the identity gate, has no effect.
- Gates can be chained together in a sequence, e.g. if an X gate is followed by an H gate, we write HX. The gate first applied should be on the right.
- Keep in mind, the order here matters! The gate HX is not the same as XH.
- For two-qubits, an arbitrary superposition is written: $|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$.

5. The Bloch sphere

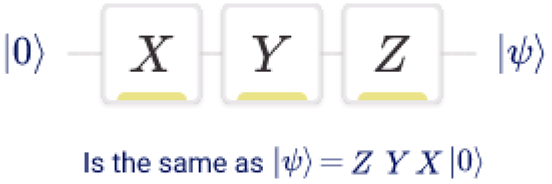
> Putting everything together, you are now ready for the Bloch sphere, which represents the full set of qubit states. You will learn how the Bloch sphere works through direct interaction. Strap in!

- Phase is what turns the Bloch circle into the Bloch sphere. The Bloch sphere represents the full set of possibilities for a qubit state.
- The Bloch sphere requires only two real numbers to specify a point on, usually represented as angles.
- As opposed to a Bloch circle, now α and β belong to a set of numbers larger than the real numbers — they can be complex numbers.. The condition which constrains states to be on the sphere is similar to that for a circle: $|\alpha|^2 + |\beta|^2 = 1$.
- A complex number encodes a distance and an angle and is written like this: $\alpha = re^{i\phi}$.



COMPLEX_NUMBER_EXPRESSION.PNG

- The equator of our Bloch sphere is a circle of radius 1. So, a point on it is a complex number specified only by a phase, ϕ.
- To incorporate complex numbers into the previous formula, the formula $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ becomes $|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle$.
- Consider the angle $\theta = \frac{\pi}{2}$, which leads to the superposition states. Now let $\phi = \frac{\pi}{2}$. The formula gives a new state with a complex phase! This state is labeled $|+i\rangle = |0\rangle + i|1\rangle$. We also have its opposing partner, i.e. $|-i\rangle = |0\rangle - i|1\rangle$.
- The phase does not affect the likelihood of ending up in $|0\rangle$ or $|1\rangle$ during measurement. The probability of obtaining a particular computational state only depends on θ.
- A common set of gates is $\{X, Y, H, Z, S, T\}$. The gates Z, S, and T are all rotations about the z axis. The Z gates is 180° or π; the S gate is 90° or $\frac{\pi}{2}$; and the T gate is 45° or $\frac{\pi}{4}$.
- A quantum circuit is a visual representation of a prescribed sequence of gates.

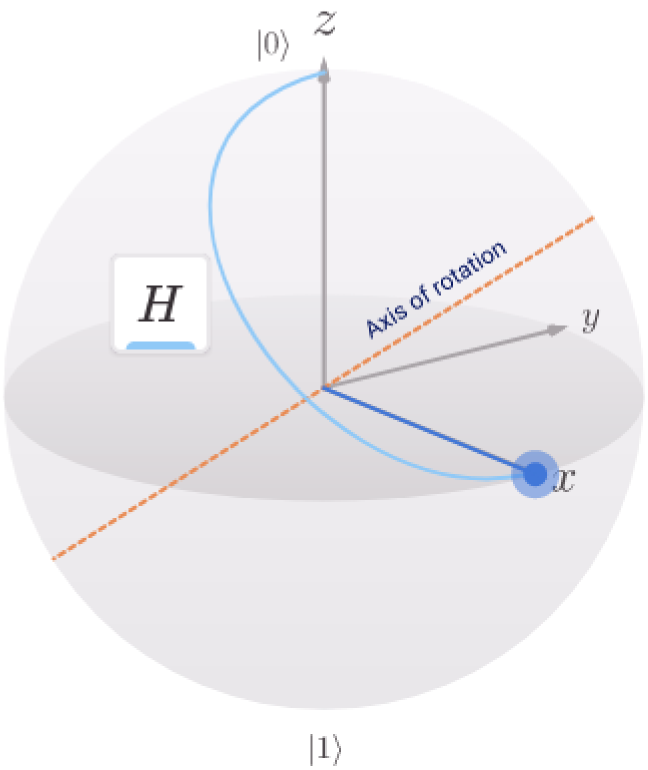


Is the same as $|\psi\rangle = Z\ Y\ X\,|0\rangle$

QUANTUM_CIRCUIT_GATES.PNG

  The qubit state is represented by a line called a wire. The gates are represented by labeled boxes that sit on the wire. Time flows from left to right — that is, the sequence of gates is applied to the initial qubit state from left to right.

- In reality, no gate is instantaneous but built by “rotating” the state on the sphere.
- The H gate is a 180° rotation about a “diagonal” axis midway between x, y, and z.



H_GATE_AXIS_ROTATION.PNG

- Full gates are those gates that have 180° rotations associated with them.

Measurement Pathway

> Measurement the destructor. What does it mean to measure something in general? What does it mean to measure something in quantum physics? Discover the dirty secrets about measurement in quantum computing.

It consists of 4 Topics:

1. Bloch party

> The concept of quantum measurement using the Bloch sphere - a visual representation of a qubit of information - is introduced. Explore measurement through interacting with a qubit.

- When a qubit is measured, the only possible results of the measurement are the computational basis states, conventionally represented by the north and south poles on the Bloch sphere.
- Quantum computations (Gates applications) are smooth rotations, but collapsing (measurement) is discontinuous and happens instantly.
- Measurement is given a special symbol in quantum circuits and typically happens at the end of an algorithm.
- The specific state of a qubit determines the probability to collapse to the north or south pole. The probability of collapsing to the north pole is a precise number $\cos\left(\frac{\theta}{2}\right)^2$. **Note** that this doesn't depend on the angle ϕ.

- The phase ϕ becomes important during computations, but not during measurements.
- We need to perform multiple measurements to get a better estimate of the state prior to measurement.
- The process of predicting what the qubit state was before a measurement was made is called tomography and is a routine diagnostic process in quantum technology labs.
- Quantum technology engineers can choose where and when a measurement happens by designing intelligent algorithms. Although most algorithms do not guarantee success, but give a high probability of success.
- Measuring at the right time is important, the time when the probability of success is highest.

2. Randomness rules

> Why are quantum measurements random and what rules govern them? Here you will understand how the symbolic notation of kets translate into real measurement outcomes and discover the Born rule.

- A generic superposition state can be written as $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$. Here, the roles of α and β are twofold. They *control the contribution of each computational state* ($|0\rangle$ and $|1\rangle$) to the superposition, and *their size tells us how likely each measurement will be*. The bigger $|\alpha|$ is, the more likely a measurement will collapse the superposition to $|0\rangle$, and similarly for β and $|1\rangle$.
- The Born rule prescribes that the probability of collapsing to $|0\rangle$ when the state upon measurement is $|\psi\rangle$ is exactly $|\alpha|^2$. Similarly, the probability of collapsing to $|1\rangle$ is $|\beta|^2$.
- The state of the qubit in terms of the Bloch sphere angles is $|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle$. A phase ϕ has no effect on the "size" of a complex number. The [Born rule] tells us that the probability of collapse to a state is only related to the angular distance from it (not the phase).
- Since it is impossible to fully control a physical qubit's environment, the qubit experiences unwanted changes in its state. We call the random couplings and fluctuations in the environment noise
- The fact that measurement collapses a qubit to a small set of possibilities (computational states) means errors are easily recognized and can be corrected.
- By encoding a single qubit of information in many physical systems we create a logical qubit whose information is spread out and protected by individual errors.

3. Get real

> Measurement in real quantum systems relevant to quantum computing: trapped ions, atoms, superconducting circuits, and more.

- We know that the qubit collapses upon measurement to either $|0\rangle$ or $|1\rangle$. But we can't perceive this as these are microscopic things. We require a way to *amplify* the results of measurements up to the scale that humans can perceive.
- A measurement of a trapped ion qubit is achieved by **state-dependent fluorescence**, which means the ion is made to give off light depending on its state. Laser beams illuminate the ion, which only scatters light if it is in the state $|1\rangle$. If no light is detected, the ion is inferred to be in the state $|0\rangle$.
- Measurement of spin qubits usually makes use of the tunneling phenomenon, whereby an electron can "escape" its trap. The chance of this happening is made to depend on what spin-state the electron occupies. Hence, the detection of a tunneling event signals that the qubit was in the more energetic state $|0\rangle$.
- In case of superconducting circuits, a "readout" antenna is integrated on the same chip near the superconducting qubit. This antenna is tuned to the appropriate frequency to receive information from the qubit. Then, the signal in the antenna is amplified until standard electronic equipment can detect it. At that point, the original state of the qubit can be inferred.
- The quantum information encoded in photons moves at the speed of light. Thus, measurement of the qubits is often passive. The photon will pass through a preset arrangement of devices and arrive at a **polarizer** which only lets through Horizontal or Vertical polarization.

4. What's the problem?

> What is the measurement problem and is it really a problem? A brief detour in metaphysics at one end of the spectrum and cold hard calculation at the other.

- Sometimes the state moves continuously (Gates), sometimes it does not (collapse). Sometimes the state moves deterministically, sometimes it does not.
- The Measurement problem is that we can't distinguish between the two rules for two different actions.
- From the computational point of view ,when we are not measuring, the information encoded by the physical system is quantum (counted in qubits), but, when we measure, the information encoded in the physical system is classical (counted in bits).
- Annoyed with the idea of our usual notion of reality only coming into existence when someone is around to collapse it, Albert Einstein famously asked “Is the Moon there when nobody looks?"
- The Copenhagen interpretation is the oldest and claims that the state is only a tool to calculate the expected results of measurements.
- The many worlds interpretation is the most famous claiming that there is no "real" collapse, and both outcomes of a quantum measurement exist in parallel worlds.

- For all the disagreement about what quantum physics is really telling us about the world and how we should think about it, everyone agrees on one thing: *it works*.
- You don't need to know what a qubit really is to perform a quantum computation.
- $HZH = X$