Qubit Pathway

> *Start with some fundamentals in how we represent information and by the end of this skill you'll understand how we can encode and process information using quantum physics!*

It also consists of 5 topics:

1. It from bit

> *This topic introduces you to the concept of information. You will learn how information is defined and represented, as well as how it is measured and used.*

- There are three fundamental types of quantitative data: analog, digital, and quantum. These data types are quantitative in that they use numbers to represent information.
  - Analog data uses real numbers, which change smoothly and continuously.
  - Digital data uses integer numbers, which can be counted and change discretely.
  - Quantum data uses complex numbers, which adds a new dimension with sometimes unintuitive consequences.
- Analog data is prone to errors while digital data entails a loss of information.
- Quantum data is the best of both worlds. While the data itself is prone to error (like analog data), with the right techniques it can also be protected against error (like digital data).
- Quantum data is single use i.e. it can not be copied. Once it is read, it collapses and is gone.
- The process of changing physical things into configurations representing data is called encoding.
- Leading candidate technologies for quantum data encoding include trapped ions or neutral atoms, superconducting circuits, defects in crystal structures, and photonic circuits.

2. Qubit from bit

> *This topic introduces quantum information. You will learn how quantum information is written and read and how it is different from digital information.*

- Whereas the state of a bit is either 0 or 1, the state of a qubit is a point on the surface of a sphere.
- Reading quantum data — a process called measurement — destroys it! The necessity of measurement to extract information from qubits means they are single-use.

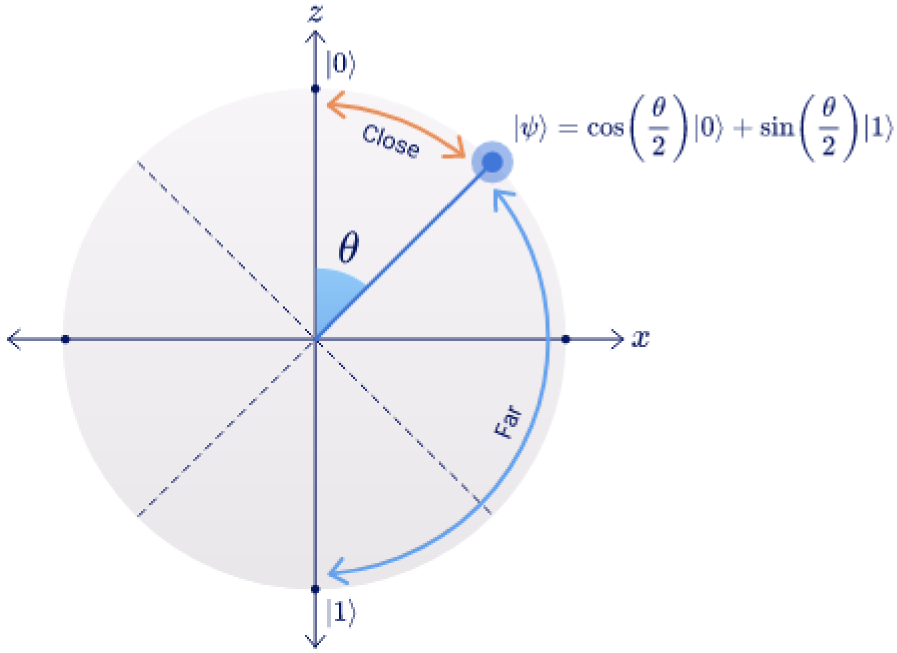
- Measurement isn't just restricted to human. Even the surrounding environment of the qubit constantly tries to alter/read the state of the qubit. This is called noise.
- Noise quickly corrupts the state of the qubit and causes errors in computation. This is the biggest challenge in quantum technology.
- The exact center of the sphere is the state of a fair coin, with an equal probability for 0 or 1 — and no quantum features! The other locations on the sphere are states of a qubit representing genuine quantum information.
- Noise randomizes the qubit's state, and sometimes, even turns it back into classical bit.
- No-gos in quantum computing are those things that we cannot do with quantum information but are possible in classical computing.
- There's no-cloning in QC refers to the fact that qubits can't be copied or cloned. Qubits of data are unique, and if you receive them, you can be assured no copies exist.
- The states on the qubit sphere that are not $|0\rangle$ and $|1\rangle$ are called superposition states. Every superposition state is a combination of $|0\rangle$ and $|1\rangle$, and can be written as $\alpha|0\rangle + \beta|1\rangle$. α and β are the constraints to keep the qubit on the sphere.
- The most common superposition state is called the **plus-state** $|+\rangle$. This is an equally weighted sum of $|0\rangle$ and $|1\rangle$: |$|+\rangle$ = $|0\rangle + |1\rangle$.
- The second most famous superposition state is the “minus-state”: $|-\rangle = |0\rangle - |1\rangle$.
- Entanglement is most succinctly defined as superpositions of correlation.
- The entire system of entangled qubits must be considered its own enormously complex state of information.
- Entanglement is necessary for quantum computation to yield an advantage over digital computation.
- In many quantum algorithms, it is assumed the qubit is supplied in the state $|0\rangle$.

3. The Bloch circle

> *You may have heard of the Bloch sphere, and we'll get there soon. Here you will find a first step towards a visualization of qubits: a circle. You will learn how a qubit is written, read, and processed in a geometric picture.*

- We can never observe superpositions as the measurement destroys superposition states and only lets us observe the computational states $|0\rangle$ and $|1\rangle$.
- This fact does not imply that the outcome of the measurement is arbitrary. The outcome of the measurement is indeed random, but occurs with a probability given by the rules of quantum mechanics.
- A circle is a geometric constraint on length
- The circular constraint, $\alpha^2 + \beta^2 = 1$, is called normalization which is required to conserve the probability. The numbers α^2 and β^2 are the probabilities, which must always add up to 1.

- The numbers α and β are called amplitudes. In classical logic we add probabilities like α^2 and β^2. In quantum logic we add amplitudes.
- We can connect the qubit superposition to the angle in our Bloch circle:



$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + \sin\left(\frac{\theta}{2}\right)|1\rangle$.

- Quantum Gates change the state a qubit from one place to another.
- For a single qubit, there are only two digital logic gates possible: the identity gate and the NOT gate.
- The convention in quantum computing is to call the NOT gate X, which exchanges 0 and 1. This works in superposition as well: $\alpha|0\rangle + \beta|1\rangle \xrightarrow{X} \alpha|1\rangle + \beta|0\rangle$.
- The Hadamard Gate H could be called the superposition gate as well since it acts as $|0\rangle \xrightarrow{H} |+\rangle$, $|1\rangle \xrightarrow{H} |-\rangle$.
- All gates have some special states they have no effect on called eigenstates, e.g. $X|+\rangle = |+\rangle$.
- The Y gate applies a 180° or π rotation about the y axis. Half a Y gate is 90° or $\frac{\pi}{2}$ rotation about the y axis.
- In addition to rotations in the clockwise direction, we can have counterclockwise y axis rotations. This is not "minus Y", but the inverse of a Y gate, Y^{-1}.

4. Do you $|ket\rangle$ it?

Kets, like |this⟩, are the abstract representation of qubits. In this brief topic, you will learn what these objects mean and how to use them.

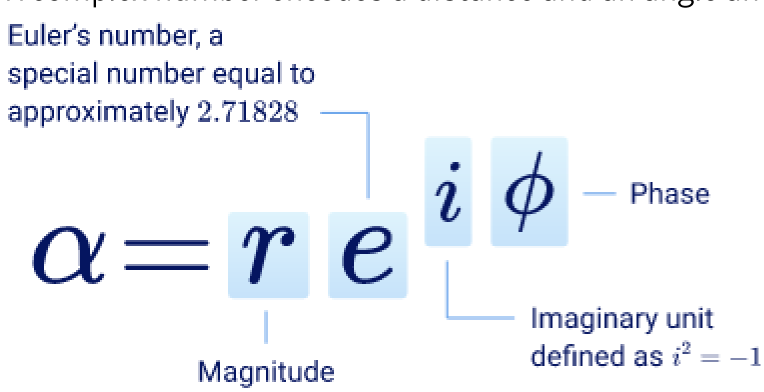
- $|\psi\rangle$ is the abstract symbol for a state of quantum information.
- Kets are symbols used to write down the states of qubits.
- In the digital world, kets are represented as vectors, e.g. $|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$.

- Any state can be written as a set of gates applied to the state $|0\rangle$.
- Gates are *applied* to qubits, i.e. a gate acts on the state written next to it on the right. The order matters.
- The symbol U is used for an arbitrary gate. So, the abstract thing we can write so far is $U|\psi\rangle$.
- Every gate U has an inverse gate, written U^{-1}. Applying U and then its inverse is written $U^{-1}U = I$. The gate I, called the identity gate, has no effect.
- Gates can be chained together in a sequence, e.g. if an X gate is followed by an H gate, we write HX. The gate first applied should be on the right.
- Keep in mind, the order here matters! The gate HX is not the same as XH.
- For two-qubits, an arbitrary superposition is written: $|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$.

5. The Bloch sphere

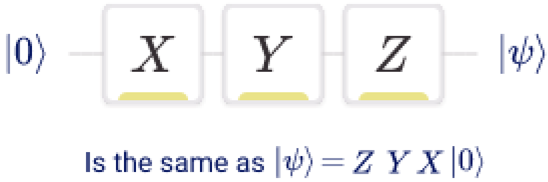
Putting everything together, you are now ready for the Bloch sphere, which represents the full set of qubit states. You will learn how the Bloch sphere works through direct interaction. Strap in!

- Phase is what turns the Bloch circle into the Bloch sphere. The Bloch sphere represents the full set of possibilities for a qubit state.
- The Bloch sphere requires only two real numbers to specify a point on, usually represented as angles.
- As opposed to a Bloch circle, now α and β belong to a set of numbers larger than the real numbers — they can be complex numbers.. The condition which constrains states to be on the sphere is similar to that for a circle: $|\alpha|^2 + |\beta|^2 = 1$.
- A complex number encodes a distance and an angle and is written like this: $\alpha = re^{i\phi}$.

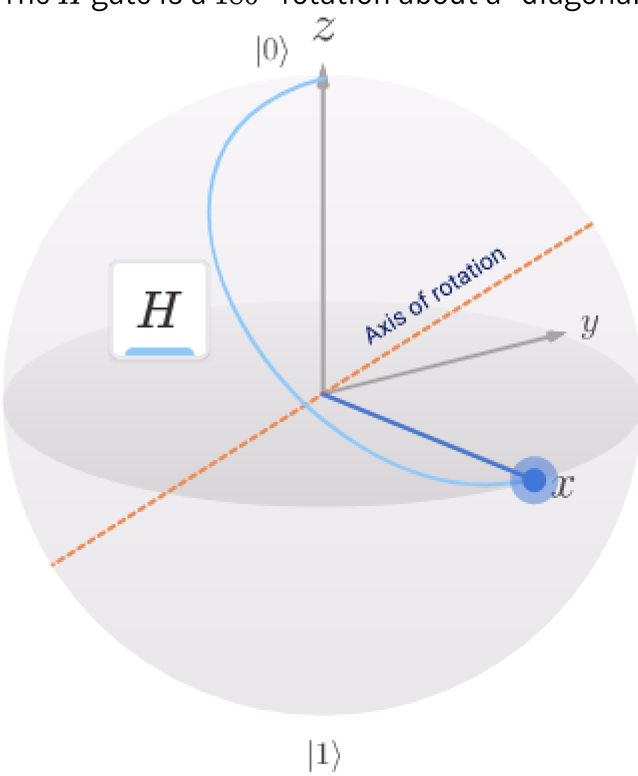


- The equator of our Bloch sphere is a circle of radius 1. So, a point on it is a complex number specified only by a phase, ϕ.
- To incorporate complex numbers into the previous formula, the formula $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ becomes $|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle$.
- Consider the angle $\theta = \frac{\pi}{2}$, which leads to the superposition states. Now let $\phi = \frac{\pi}{2}$. The formula gives a new state with a complex phase! This state is labeled $|+i\rangle = |0\rangle + i|1\rangle$. We also have its opposing partner, i.e. $|-i\rangle = |0\rangle - i|1\rangle$.

- The phase does not affect the likelihood of ending up in $|0\rangle$ or $|1\rangle$ during measurement. The probability of obtaining a particular computational state only depends on θ.
- A common set of gates is $\{X, Y, H, Z, S, T\}$. The gates Z, S, and T are all rotations about the z axis. The Z gates is 180° or π; the S gate is 90° or $\frac{\pi}{2}$; and the T gate is 45° or $\frac{\pi}{4}$.
- A quantum circuit is a visual representation of a prescribed sequence of gates.



The qubit state is represented by a line called a wire. The gates are represented by labeled boxes that sit on the wire. Time flows from left to right — that is, the sequence of gates is applied to the initial qubit state from left to right.
- In reality, no gate is instantaneous but built by "rotating" the state on the sphere.
- The H gate is a 180° rotation about a "diagonal" axis midway between x, y, and z.



- Full gates are those gates that have 180° rotations associated with them.